Content: Math	**Grade/Course**: 3rd Grade	**Timeline**: Week 1

Standard(s): 3.OA.2 Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.

Lesson Overview:	**Lesson Objective(s)**:
Students' mastery of this skill will include an understanding of two types of contexts that can be represented using division. One is that (for example) 56 divided by 8 is the number of objects in each share when 56 objects are divided into 8 equal shares. The other is that 56 divided by 8 is the number of shares when 56 objects are partitioned into shares of 8 objects each. These two types of division contexts are often described as "partitive" and "quotative", respectively. When constructing division problems, teachers often lean heavily towards the former; e.g., "If I have 15 cookies and want to divide them among five children, how many cookies can each child have?" It is important that teachers also introduce students to the other type of division context; e.g., "If I have 15 cookies and want to make gift bags of 5 cookies each, how many gift bags can I make?" Students should describe a context in which a number of shares or a number of groups can be expressed as a division statement. Further, students will eventually make the connection between division and multiplication.	I Can Statements: • I can explain division as a set of objects partitioned into an equal number of shares. • I can identify parts of division equations (dividend, divisor, and quotient) • · I can interpret quotients in division (32 ÷ 4 = 8 can be 4 groups with 8 items in each group or 8 groups with 4 items)
Vocabulary: dividend, division, divisor, equal shares, groups, partition, quotient, equation, sharing, unknown/symbol, inverse relationship	**Focus Question(s)**: What strategies can I use to help me understand mathematical problems involving division? How do properties help to divide? What is the relationship between multiplication and division?

Description of Lesson (Including Instructional Strategies):

Anticipatory Set:

Concept building: What is division? Show the online demo lesson on Learnzillion:

https://learnzillion.com/resources/72522-interpret-whole-number-quotients-of-whole-numbers-3-oa-a-2

Instruction and Strategies:

- Start with a real-world problem such as: "I have eight kids who want to play volleyball. I need two teams of equal players. How many will be on each team? How should I 'divide' them up?" Discuss the problem with your students, then show them the equation on the board for this problem: 8/2=4. You can also draw a picture of something like twelve cupcakes and four friends, and show how the problem 12/4=3 cupcakes for each person.

- Show the fact family to go with the real-world problems and equations you discussed in Step 2. Once they understand what division means, you can show them how to figure out the answer quickly. For example, if students know that 4x3 =12, then they can see the relationship between 12/4=3 and the multiplication problem.

Give students a problem such as 20/4=?, and then ask students to solve this problem to find the quotient: 4x?= 20. Explain that whatever numeral replaces the ? in the equation (in this case 5) is the answer to the division problem.

- Discuss how some division problems will not always have a whole number for the answer. For example, give your students a realistic problem such as, "I have 22 cookies and 5 cookies can fit in each box. How many full boxes will I have, and how many left over cookies?" Then show the equation 22/5=?. Work on the problem together with illustrations or even manipulatives until students figure out the answer 4 boxes with 2 cookies left over or 4 R 2. Again, make sure students understand what the R stands for (remainder) and what remainder means. It is important for students to make connections with multiplication and real-life applications when learning division.

Notes:

- Students develop an understanding of the meanings of multiplication and division of whole numbers through activities and problems involving equal-sized groups, arrays, and area models; multiplication is finding an unknown product, and division is finding an unknown factor in these situations.
- The use of drawings, pictures, and manipulative objects will aid in students' understanding of division as the process of making equal shares from a whole number.
- Common contexts should be used that easily lend themselves to whole number quotients that are typically partitioned into equal shares. For example, if you are packaging bottles of juice for an event into 6-packs, and you have 42 bottles, how many 6-packs do you have? Another example: how many dozens of eggs are there in 132 eggs? 132 ÷ 12 = 11. A third example: 42 ÷ 6 = 7 .49 days is how many weeks? 49 ÷ 7 = 7.

Guided Practice:

- Work on 3 word problems with the students as they solve on their personal white boards. Use Active Participation Strategies to check for understanding. (Think-Pair-Share, Partner Work, Thumbs Up-Thumbs Down, etc.)

Formative Assessment:

- Have the students work on 3 word problems at their desks with their partners.

Independent Practice:

- Quick Check/Exit Ticket

Closure:

Lesson Debrief:

How does knowledge of multiplication help you divide?

Accommodations/Modifications:

Model the steps in a small group. Show the multiplication table for visual support.

Resources (Textbook and Supplemental): Guam District Curriculum Guide, Howard County Common Core Math, Read Tennessee, 3rd Grade Math Flipbook

Reflection:

Content: Math	**Grade/Course**: 3rd Grade	**Timeline**: Week 1
Standard(s): 3.OA.3 Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.		

Lesson Overview: Students will draw objects in arrays and equal groups and write equations to solve division word problems. They will explain the connection between their drawing and the equations to the division word problems.	**Lesson Objective(s)**: I Can Statements: • I can justify in words the relationship between a drawing and the equation that are used to solve a division word problem.
Vocabulary: partition division, measurement division, distribution, division, equal arrays, equal groups	**Focus Question(s)**: How does drawing a picture and writing an equation help you solve a division word problem?

Description of Lesson (Including Instructional Strategies):

Anticipatory Set:

Play the video of the lesson from Learnzillion as a springboard/preview of the lesson:

https://learnzillion.com/resources/72645-solve-multiplication-and-division-word-problems-3-oa-a-3

Instruction and Strategies:

- After playing the video, work a problem with the whole class, referring to the strategies learned in the video to help visualize the problem. Use Think-Pair-Share to ask about each step or strategy that they learned.

- Have the students write the "formula" for visualizing equal groups: # of groups x #in each group = total
  This is important for them to understand that both multiplication and division word problems deal with equal groups. This will also help them identify what is the unknown in either a multiplication or division word problem. Moreover, such "formula" will help them understand the inverse relationship between multiplication and division. (Concept building)

Guided Practice:

- Students can begin to practice similar problems in pairs using items #3–5 in the Equal Groups worksheet. Facilitate to ensure that students are explaining the relationship between their drawing and the equation.
- Students will record their justification on the relationship between their drawing and equation

Formative Assessment:

- Fingers-Up strategy: Students will indicate their degree of understanding by using a 1, 2, 3 scale. (1: Don't understand, 2: Some understanding, but need more clarification, and 3: Got it!) Use this strategy as you ask students about specific word problems as you are moving through this lesson.

Independent Practice:

- Quick Check and Homework

Closure:

Application Cards—Students think of a way to apply their new knowledge or skill in the real world and write it down on an index card. Collect the cards and either share them anonymously with the class or keep them to review privately.

Accommodations/Modifications:
Provide students with manipulatives to represent groups. Model how to use the manipulatives to represent groups by separating counters into different number of groups in various sets. Peer modeling will be used to show students how to visualize and illustrate division equations on paper.

Resources (Textbook and Supplemental): Guam District Curriculum Guide, learnzillion.com, Howard County Common Core Math, Read Tennessee, 3rd Grade Math Flipbook Teacher-made practice worksheet, flow chart, manipulatives

Reflection:

Content: Math	**Grade/Course**: 3rd Grade	**Timeline**: Week 2
Standard(s): 3.OA.4 Determine the unknown whole number in a division equation relating three whole numbers.		

Lesson Overview: As students gain more practice and fluency with multiplication and division problems, they can begin to relate three whole numbers. For example, determine the unknown number that makes the equation true in each of the equations: 8 x ? = 48; 5 = ? ÷ 3; 6 x 6 = ?	**Lesson Objective(s)**: I Can Statements: • I can determine the unknown number in division problems such as in the following examples: 8 x 9 = ?, 8 x ? = 48, ? x 3 = 27 • I can identify the parts and whole of an equation • I can identify symbols used for missing numbers\ .
Vocabulary: equal sign, multiplication equation, division equation, unknown variable	**Focus Question(s)**: What multiplication or division strategy might apply to a situation? How do you determine that the strategy worked?

Description of Lesson (Including Instructional Strategies):

Anticipatory Set:

Teacher reminds students of how math problems can be represented as a number sentence where students must find the value of a missing number in the middle; for example, subtraction can be represented as an addition number sentence with a missing addend, and division can be represented as a multiplication number sentence with a missing factor, but the concept can manifest itself in other problems, too.

Examples:
$8 \times x = 32$
$17 - x = 15$

Instruction and Strategies:

- Online Demo Lesson: https://learnzillion.com/resources/72854-determine-unknown-whole-numbers-in-multiplication-or-division-equations-3-oa-a-4

- A connection to 3.OA.4 is made to extend beyond the traditional notion of fact families, by having students explore the inverse relationship of multiplication and division. Students solve problems and determine unknowns in equations. Students should also experience creating story problems for given equations. When crafting story problems, they should carefully consider the question(s) to be asked and answered to write an appropriate equation.
- Students apply their understanding of the meaning of the equal sign as "the same as" to interpret an equation with an unknown.

  When given 4 x ? = 40, they might think:
  - 4 groups of some number is the same as 40.
  - 4 times some number is the same as 40.
  - I know that 4 groups of 10 is 40, so the unknown number is 10.
  - The missing factor is 10 because 4 times 10 is equal to 40.
  - Equations in the form of a x b = c and c = a x b should be used interchangeably, with the unknown in different positions.

  Examples:
  - Solve the equation: 24 = ? x 6

- Rachel has 3 bags. There are 4 marbles in each bag. How many marbles does Rachel have altogether? (3 x 4 = m)

Guided Practice:

- Work on 3 word problems with the students as they solve on their personal white boards. Use Active Participation Strategies to check for understanding. (Think-Pair-Share, Partner Work, Thumbs Up-Thumbs Down, etc.)

Formative Assessment:

- Have the students work on 3 problems at their desks independently first, then have them compare answers with their partners.

Independent Practice:

- Quick Check/Exit Ticket

Closure:
Lesson Debrief:
What strategies did you use to solve for the unknown number?
How did you determine that your strategy worked?

Accommodations/Modifications:
Model the steps in a small group to provide more focused instruction. Use manipulatives and models.

Resources (Textbook and Supplemental): Guam District Curriculum Guide, Howard County Common Core Math, Read Tennessee, 3rd Grade Math Flipbook

Reflection:

Content: Math	**Grade/Course**: 3rd Grade	**Timeline**: Week 2
Standard(s): 3.OA.5 Apply properties of operations as strategies to multiply and divide.		

Lesson Overview:	**Lesson Objective(s)**:
Just like in addition and subtraction, students need a variety of ways to solve for number sentences involving multiplication and division without relying on rote memorization. Students should employ a variety of solution methods that demonstrate their understanding of the meaning of multiplication. These methods include the use of the commutative property of multiplication, the associative property of multiplication, and the distributive property of multiplication over addition.	I Can Statements: • I can explain the commutative, associative, and distributive property of multiplication. • I can apply the commutative, associative, and distributive properties to decompose, regroup, and/or reorder factors to make it easier to multiply two or more factors.
Vocabulary: equal sign, multiplication equation, unknown variable, commutative, distributive, associative property	**Focus Question(s)**: What multiplication strategy might apply to a situation? How do you determine that the multiplication strategy worked?

Description of Lesson (Including Instructional Strategies):

Anticipatory Set:

Present this problem to the class:

Kyshawn was working on his homework when he came to a multiplication problem that he hasn't learned yet. The problem was 6 x 5 = _______. Kyshawn has learned some multiplication facts. He knows that 2 x 5 = 10 and 4 x 5 = 20. How can Kyshawn use what he knows to find the solution to his homework problem? Use drawings, words, or equations to explain.

Call on volunteers to show how they figured out the answer.

Instruction and Strategies:

- Online Demo Lesson: https://learnzillion.com/resources/72372-apply-properties-of-operations-as-strategies-to-multiply-and-divide-3-oa-b-5

- Students should have a variety of opportunities to work with number sentences that are missing one of the whole numbers.
- Continued work with multiplication and division fact families will aid students in finding solutions much faster.
- Students should learn how to solve problems using the following strategies:
  - Example: If 6 x 4 = 24 is known, then 4 x 6 is also known (commutative property of multiplication).
  - Example: 3 x 5 x 2 can be found by 3 x 5 = 15, then 15 x 2 = 30, or by 5 x 2 = 10, then 3 x 10 = 30 (Associative property of multiplication).
  - Example: Knowing that 8 x 5 = 40 and 8 x 2 = 16, one can find 8 x 7 as 8 x (5 + 2) = (8 x 5 + (8 x 2) - 40 + 16 + 56 (Distributive porperty of multiplication).
- While students DO NOT need to use the formal terms of these properties, they should understand that properties are rules about how numbers work; students do need to be flexible and fluent in applying each of them.
- Students represent expressions using various objects, pictures, words, and symbols in order to develop their understanding of properties. They multiply by 1 and 0 and divide by 1. They change the order of numbers to determine that the order of numbers does not make a difference in multiplication (but does make a difference in division). Given three factors, they investigate changing the order of how they multiply the numbers to

determine that changing the order does not change the product. They also decompose numbers to build fluency with multiplication. (Marzano: Generating and Testing Hypothesis)
- The associative property states that the sum or product stays the same when the grouping of addends or factors is changed. For example, when a student multiplies 7 x 5 x 2, a student could rearrange the numbers to first multiply 5 x 2 = 10 and then multiply 10 x 7 = 70.
- The commutative property (order property) states that the order of numbers does not matter when adding or multiplying numbers. For example, if a student knows that 4 x 3 = 12, then they also know that 3 x 4 = 12. The array below could be described as a 4 x 3 array for 4 rows and 3 columns or 3 rows and 4 columns. There is no "fixed" way to write the dimensions of an array as rows x columns or columns x rows.
- Students are introduced to the distributive property of multiplication over addition as a strategy for using products they know to solve products they don't know. Here are ways that students could use the distributive property to determine the product of 7 x 6. Again, students should use the distributive property, but can refer to this in informal language such as "breaking numbers apart."

Guided Practice:
- Work on 3 word problems with the students as they solve on their personal white boards. Use Active Participation Strategies to check for understanding. (Think-Pair-Share, Partner Work, Thumbs Up-Thumbs Down, etc.)

Formative Assessment:
- Have the students work on 3 problems at their desks independently first, then have them compare answers with their partners.

Independent Practice:
- Quick Check/Exit Ticket

Closure:
Lesson Debrief:
How can you illustrate each property?
How did you determine which property will work?

Accommodations/Modifications:
Model the steps in a small group to provide more focused instruction. Use manipulatives and models.

Resources (Textbook and Supplemental): Learnzillion.com, Guam District Curriculum Guide, Howard County Common Core Math, Read Tennessee, 3rd Grade Math Flipbook

Reflection:

Content: Math	**Grade/Course**: 3rd Grade	**Timeline**: Week 3

Standard(s): 3.OA.7 Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8× 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.

Lesson Overview:	**Lesson Objective(s)**:
As students become more sophisticated with their knowledge of multiplication and division, they develop fluency to solve a wide variety of multiplication and division problems.	• I can multiply any two numbers with a product within 100 with ease by picking and using strategies that will get to the answer fairly quickly. • I can instantly recall from memory the product of any two one-digit numbers.
Vocabulary: product, equal sign, multiplication equation, unknown	**Focus Question(s)**: What is multiplication? What multiplication strategy might apply to a situation?

Description of Lesson (Including Instructional Strategies):

Anticipatory Set:

Review on interpreting multiplication equations:

Have the students model the following equations: 5 x 6 8 x 3 4 x 7

They can use equal groups, array, area of a rectangle, jumps on a number line or repeated addition.

Have selected students present their work.

Review: What does the symbol x mean? What does the = sign mean?

Instruction and Strategies:

- Online Resources: https://learnzillion.com/lesson_plans/5649-understand-multiplication-with-0-and-1 https://hcpss.instructure.com/courses/97/modules/items/22263

- "Know from memory" does not mean focusing only on timed tests and repetitive practice, but ample experiences working with manipulatives, pictures, arrays, word problems, and numbers to internalize the basic facts (up to 9 x 9). The CCSS define the word *fluently* as accuracy, efficiency (using a reasonable amount of steps and time), and flexibility (using strategies such as the distributive property). Instruction should focus on building fluency over time. (Marzano: Reinforcing Effort and Providing Recognition)

- Strategies students may use to attain fluency include:
  Multiplication by zeros and ones
  Doubles (2s facts), Doubling twice (4s), Doubling three times (8s)
  Tens facts (relating to place value, 5 x 10 is 5 tens or 50)
  Five facts (half of tens)
  Skip counting (counting groups of __ and knowing how many groups have been counted)
  Square numbers (ex: 3 x 3)
  Nines (10 groups less one group, e.g., 9 x 3 is 10 groups of 3 minus one group of 3)
  Decomposing into known facts (6 x 7 is 6 x 6 plus one more group of 6)
  Turn-around facts (Commutative Property)
  Fact families (Ex: 6 x 4 = 24; 24 ÷ 6 = 4; 24 ÷ 4 = 6; 4 x 6 = 24)
  Missing factors

- General Note: Students should have exposure to multiplication and division problems presented in both vertical and horizontal forms.

- Note that mastering this material and reaching fluency in single-digit multiplications and related divisions with understanding may be quite time-consuming because there are no general strategies for multiplying or dividing all single-digit numbers as there are for addition and subtraction. Instead, there are many patterns and strategies dependent upon specific numbers. So it is imperative that extra time and support be provided if needed. Such fluency may be reached by becoming fluent for each number (e.g., the 2s, the 5s, etc.) and then extending the fluency to several, then all numbers, mixed together. Organizing practice so that it focuses most heavily on understood but not yet fluent products and unknown factors can speed learning.

Guided Practice:
- Flashcard Drill with teacher, then with partners
- Multiplication Fact Games

Formative Assessment:
- Timed Test for Mastered Tables
- Worksheet Practice

Independent Practice:
- Computer-assisted Multiplication Practice
- Writing Facts Repeatedly

Closure:
Sit in a circle. Call on some volunteers to recite their favorite Times Table. Do a quick review of the times table focus of the day.

Accommodations/Modifications:
Provide the multiplication table as visual aid.

Resources (Textbook and Supplemental): Guam District Curriculum Guide, www.readtennessee.org

Reflection:

Content: Math	Grade/Course: 3rd Grade	Timeline: Week 3

Standard(s): 3.OA.8 Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

Lesson Overview:	**Lesson Objective(s)**:
This lesson is focused on developing strategies to solve two-step word problems. Students need to be exposed to multiple problem-solving strategies (using any combination of words, models, numbers, diagrams, physical objects, or symbols) and be able to choose which ones to use. The size of the numbers should be limited to related third grade standards.	I Can Statements: • I can choose the correct operation to perform the first computation, and • I can choose the correct operation to perform the second computation in order to solve two-step word problems. • I can write equations using a letter for the unknown number. • I can decide if my answers are reasonable using mental math and estimation strategies including rounding.
Vocabulary: addition, estimation strategies, mental computation, commutative property over addition, associative property over addition, rounding, subtraction unknown variable	**Focus Question(s)**: What are different ways to represent a given problem? What multiplication strategy might apply to a situation?

Description of Lesson (Including Instructional Strategies):

Anticipatory Set:
Present a two-step problem to the students. Challenge the class to solve it on their own. Then have them share their work with an elbow partner. Call on several students who used different strategies to present their solutions to the class using the document camera (if available). Then use the problem as a springboard to introduce the lesson and highlight how a problem can be solved using different strategies.

Instructional Strategies:

- **Online Resource:**
  How to teach the standard**:** https://learnzillion.com/lesson_plans/8632-solving-two-step-word-problems-using-a-model#fndtn-lesson

- Students gain a full understanding of which operation to use in any given situation through contextual problems. Number skills and concepts are developed as students solve problems.
- The use of pictures and diagrams is an important tool to aid students with problem-solving and is also useful for justifying a particular answer.
- Problems should be presented on a regular basis as students work with numbers and computations. (Researchers and mathematics educators advise against providing “key words” for students to look for in problem situations because they can be misleading.)
- Students should use various strategies to solve problems. Students should analyze the structure of the problem to make sense of it. They should think through the problem and the meaning of the answer before attempting to solve it. (CITW: Identifying Similarities and Differences)
- Encourage students to represent the problem situation in a drawing or with counters or blocks. Students should determine the reasonableness of the solution to all problems using mental computations and estimation strategies. (CITW: Nonlinguistic Representations)
- Begin with word problems that promote more than one way to solve and encourage students to justify their thinking and be able to explain someone else’s way of solving the problem.

- When students solve word problems, they should use various estimation skills, which include identifying when estimation is appropriate, determining the level of accuracy needed, selecting the appropriate method of estimation, and verifying solutions or determining the reasonableness of solutions. (CITW: Generating and Testing Hypotheses)
- Estimation strategies include, but are not limited to:
    - Using benchmark numbers that are easy to compute
    - Front-end estimation with adjusting (using the highest place value and estimating from the front end making adjustments to the estimate by taking into account the remaining amounts)
    - Rounding and adjusting (students round down or round up and then adjust their estimate depending on how much the rounding changed the original values)

Formative Assessment:
- Students solve a given problem on their personal white boards
- Partner Practice/Team Huddle
- Quick Check

Independent Practice:
- Give several problem for independent practice.

Closure:
Have the students answer the focus questions after each lesson.

Accommodations/Modifications:
For students who need intensive support, give simpler problems with lower numbers.

Resources (Textbook and Supplemental): Guam District Curriculum Guide, Engage NY, Howard County, About Education.com, Read Tenessee, 3rd Grade Math Flipbook, http//www.learnzillion.com

Reflection:

Content: Math	**Grade/Course**: 3rd Grade	**Timeline**: Week 4

Standard(s):
3.MD.7.b Relate area to the operations of multiplication and addition. b. Multiply side lengths to find areas of rectangles with whole- number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.
3.MD.7.c Relate area to the operations of multiplication and addition. c. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and $b + c$ is the sum of $a \times b$ and $a \times c$. Use area models to represent the distributive property in mathematical reasoning.
3.MD.7.d Relate area to the operations of multiplication and addition. d. Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems.

Lesson Overview:
Once students have mastered the concept of area, they are ready to make calculations with arithmetic operations. This standard has many parts: Students should be able to find the areas of rectangles by multiplication and know why the area of a rectangle with whole-number side lengths is the product of the side lengths (because an *m*-unit by *n*-unit is composed of *m* x *n* array of unit squares, which tile the rectangle without overlapping, thus the area of the rectangle is *m* times *n*). They should then be able to represent an arbitrary product of whole numbers using an area model, and to use the model to understand distributive property of multiplication. Finally, students should be able to find areas of rectilinear (characterized by straight lines) figures by decomposing them into rectangles.

Lesson Objective(s):
I Can Statements:

- I can use tiles to find the area of rectangles.
- I can explain the relationship between tiling and multiplying side lengths to find the area of rectangles.
- I can multiply adjacent side lengths of rectangles to solve word problems.
- I can use area models to explain the distributive property.
- I can decompose an irregular figure into non-overlapping rectangles.
- I can explain area as additive and use this understanding to solve word problems.

Vocabulary: area, closed figure, length, rectangular array,width, decompose, irregular shape, rectilinear figure, square units, two-dimensional

Focus Question(s):
What properties between additive and distributive do I use in order to find the area?
How can students explain the relationship of the distributive property in context using pictures, words, and numbers to support their reasoning?

Description of Lesson (Including Instructional Strategies):
Anticipatory Set:

Give students a rectangle with whole number side lengths and ask them to predict the area. Then ask them to multiply the side lengths, then tile the rectangle with unit squares. Ask them to explain why these two are the same.

Instruction and Strategies:

- Online Demo Lessons:
  https://learnzillion.com/resources/73010-relate-area-to-the-operations-of-multiplication-and-addition
  https://learnzillion.com/resources/72595-recognize-area-as-additive-find-area-of-figures-by-decomposing-them-3-md-c-7d
  https://learnzillion.com/resources/72499-relate-area-to-multiplication-and-addition-using-unit-squares-and-arrays-3-md-c-7a-3-md-c-7b

- Students should be moving on to problems in which they find the area of larger rectangles by "seeing" the array of unit squares within a rectangle without having to physically create it.

- Ask students to explain multiplication to someone else using an area model. Ask students whether they can find a shortcut to multiplying *a* and *b*, multiplying *a* and *c*, and adding the two products; for example, instead of computing 7 x 4 + 7 x 6 by computing two separate products and adding the results, recognize that this is 7 x 10 = 70. Have them interpret this problem using an area model. Ask students to come up with a mathematical justification of the distributive property.

- Area is additive; perhaps ask students to compute the area of a figure that is made by adjoining two rectangles, and then help them see that the area of this figure is the sum of the areas of the two rectangles (and can be gotten by multiplying the side lengths of each rectangle, and then adding the products).

- Using concrete objects or drawings, students build competence with composition and decomposition of shapes, spatial structuring, and addition of area measurements. Students learn to investigate arithmetic properties using area models. For example, they learn to rotate rectangular arrays physically and mentally, understanding that their areas are preserved under rotation, and thus, for example, 4 x 7 = 7 x 4, illustrating the commutative property of multiplication. Students also learn to understand and explain that the area of a rectangularregion of, for example, 12 length-units by 5 length-units can be found either by multiplying 12 x 5, or by adding two products, e.g., 10 x 5 and 2 x 5, illustrating the distributive property as in the previous example.

Guided Practice:
- Have the students work on two or three problems on their personal boards, then compare their answers with their elbow partners then with their teams.

Formative Assessment:
- Worksheet on finding the area of rectangles and rectilinear areas.

Independent Practice:
- Performance Task:

1. Give each student square manipulatives or 2 cm graph paper (varied sized will also work). Then give each student an area amount that can be made in multiple ways.
2. Have students either color in the graph paper or use the square manipulatives to create the area given in as many ways as possible.
3. Once students have built the different area models of the given number, have them explain their strategy.

Closure:
Lesson Debrief:
Have the students answer the focus questions.

Accommodations/Modifications:
Provide the multiplication table for Day 3. Work with a small group who needs extra support.

Resources (Textbook and Supplemental): Guam District Curriculum Guide, Howard County Common Core Math, Read Tennessee, 3rd Grade Math Flipbook

Reflection:

Content: Math	**Grade/Course**: 3^{rd} Grade	**Timeline**: Weeks 6 and 7

Standard(s):

3.NF.1 Understand a fraction 1/*b* as the quantity formed by 1 part when a whole is partitioned into *b* equal parts; understand a fraction *a*/*b* as the quantity formed by *a* parts of size 1/*b*.
3.NF.2a Understand a fraction as a number on the number line; represent fractions on a number line diagram. a. Represent a fraction 1/*b* on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into *b* equal parts. Recognize that each part has size 1/*b* and that the endpoint of the part based at 0 locates the number 1/*b* on the number line.
3.NF.2b Understand a fraction as a number on the number line; represent fractions on a number line diagram. b. Represent a fraction *a*/*b* on a number line diagram by marking off *a* lengths 1/*b* from 0. Recognize that the resulting interval has size *a*/*b* and that its endpoint locates the number *a*/*b* on the number line.

Lesson Overview:

3.NF.1 Grade 3 expectations in the two fraction standards are limited to fractions with denominators 2, 3, 4, 6, 8. Students develop an understanding of fractions, beginning with unit fractions. Students view fractions in general as being built out of unit fractions, and they use fractions along with visual fraction models to represent parts of a whole. Students understand that the size of a fractional part is relative to the size of the whole. Students use their understanding of fractions on the number line to use a ruler to measure lengths to halves and quarter units.

3.NF.2 Up until this point, students have had very limited exposure to fractions. Now they will work with numbers that exist between two whole numbers. Students learn that a fraction is not only a "part of a whole", but also a point on the number line. This is significant because students often think of fractions (such as 3/8) as objects in a completely separate category from whole numbers. Locating the fraction 3/8 on a number line shows that a fraction can be thought of as between two whole numbers.

Lesson Objective(s):
I Can Statements:

- I can explain any unit fraction as one part of a whole.
- I can explain any fraction (a/b) as "a" (numerator) being the numbers of parts and "b" (denominator) as the total number of equal parts in the whole.
- I can represent a fraction and explain my representation.
- I can explain and show how 1/b can be represented on a number-line in two ways (1) as a number that is located a distance of 1/b to the right of 0, and (2) as the size of each of the parts when a whole is partitioned into b equal parts.
- I can explain and show how a/b can be represented on a number line in two ways: (1) as a number that is located a distance a/b to the right of 0, and (2) as the size of a parts when a whole is partitioned into b equal parts.
- I can represent a unit fraction (1/b) on a number line between 0 and 1.
- I can represent any fraction (a/b) on a number line.

Vocabulary: denominator, equal parts, fair shares, fraction, numerator, one-eighth; 1/8, one-fourth; ¼, one-half; ½, one-sixth; 1/6, one-third; 1/3, partitioned, unit fraction, whole, divide, fractional parts, number line diagram, numerator

Focus Question(s):
How will I show a fraction using concrete models?
What does the number in a numerator mean?
How do you partition parts of a whole to represent a fraction?
How do you partition a number line to represent a fraction?

Description of Lesson (Including Instructional Strategies):
Anticipatory Set:

Show a picture or a drawing of a pizza. Pose a problem:

If four friends want to share the pizza equally, how would you cut the pizza?
What do you call the part that each person gets after dividing it equally?
Use this problem to present the word *fraction* and what it means.

Instruction and Strategies:

- Online Resource:
  https://learnzillion.com/resources/72250-understand-fractions
  https://learnzillion.com/resources/72626-represent-fractions-on-a-number-line-3-nf-a-2

- The fraction standards should refer to the sharing of a whole being partitioned or split. Fraction models in third grade include area (parts of a whole), models (circles, rectangles, squares), and number lines. Some representations, such as a rectangle, are easier to partition, than say, a circle. Use various contexts familiar to students, such as candy bars, fruit, and cakes. Set models, another type of modeling (parts of a group), are not explored in the third grade curriculum. In 3.NF.1, students should focus on the concept that a fraction is made up (composed) of many pieces of a unit fraction, which has a numerator of 1. For example, the fraction 3/5 is composed of 3 pieces in that each of the three pieces has a size of 1/5. (Marzano: Cooperative Learning, Identifying Similarities and Differences, Nonlinguistic Representations)
- Some important concepts related to developing understanding of fractions include:
  - Understand fractional parts must be equal-sized
  - The number of equal parts tells how many make a whole (the denominator).
  - As the number of equal pieces in the whole increases, the size of the fractional pieces decreases (1/6 is smaller than 1/3); this is often confusing to students. They see larger whole numbers as bigger numbers and want to assume the same is true for fraction denominators; emphasize by partitioning the same size whole into different pieces for comparison. For the same size rectangular cake, ask: “If you like cake, which piece would you prefer, 1/10 or 1/6?” Have students make a conjecture about unit fractions and relative sized denominators.
  - The size of the fractional part is relative to the whole. The number of children in one half of a classroom is different than the number of children in one-half of a school. (The whole in each set is different, therefore the half in each set will be different.) Another example: 1/2 of the paint in a small bucket could be less paint than 1/3 of the paint in a larger bucket, but 1/3 of a ribbon is longer than 1/5 of the same ribbon because when the ribbon is divided into 3 equal parts, the parts are longer than when the same length ribbon is divided into 5 equal parts.
  - When a whole is cut into equal parts, the denominator represents the number of equal parts. (A whole is cut into four same size parts, the function of the denominator; each part represents 1/4 of the whole.) Initially, students can use an intuitive notion of “same size and same shape” (congruence) to explain why the parts are equal. For example, when they divide a square into four equal squares or four equal rectangles. Students come to understand a more precise meaning for “equal parts” as “parts with equal measurements.” For example, when a ruler is partitioned into halves or quarters of an inch, they see that each subdivision has the same length. In area models, they reason about the area of a shaded region to decide what fraction of the whole it represents.
  - The numerator of a fraction is the counting number of equal parts. Therefore, ¾ means that there are 3 one-fourths. Students should count as they would for wholenumbers, 1, 2, 3, . . . but now count fraction parts of a whole: one-fourth, two-fourths, three-fourths . . .
  - Given a shape, students partition it into equal parts, recognizing that these parts all have the same area. They identify the fractional name of each part and are able to partition a shape into parts with equal areas in several different ways.
- Activity: Students think all shapes can be divided the same way. Present shapes other than circles, squares, or rectangles to prevent students from overgeneralizing that all shapes can be divided the same way. (Marzano: Nonlinguistic Representations) For example, have students fold a triangle into eighths. Provide oral directions for folding the triangle:
  - Fold the triangle in half by folding the left vertex (at the base of the triangle) over to meet the right vertex.
  - Fold in this manner two more times.
  - Have students label each eighth using fractional notation. Then, have students count the fractional parts in the triangle (one-eighth, two-eighths, three-eighths, and so on).

- Students transfer their understanding of parts of a whole to partition a number line into equal parts. There are two new concepts addressed in this standard that students should have time to develop.
- On a number line from 0 to 1, students can partition (divide) it into equal parts and recognize that each segmented part represents the same length.

Guided Practice:
- Activity: Students think all shapes can be divided the same way. Present shapes other than circles, squares, or rectangles to prevent students from overgeneralizing that all shapes can be divided the same way. (Marzano: Nonlinguistic Representations) For example, have students fold a triangle into eighths. Provide oral directions for folding the triangle:
    - Fold the triangle in half by folding the left vertex (at the base of the triangle) over to meet the right vertex.
    - Fold in this manner two more times.
    - Have students label each eighth using fractional notation. Then, have students count the fractional parts in the triangle (one-eighth, two-eighths, three-eighths, and so on).

Formative Assessment:
- White Board Exercises
- Worksheets on identifying fractions from given shapes and on the number lline (with and without partner)

Independent Practice:
- Quick Checks
- Worksheets
- Homework

Closure:
Lesson Debrief:
Have the students answer the focus questions.

Accommodations/Modifications:
Provide manipulatives and fraction charts and number lines on the wall for reference.

Resources (Textbook and Supplemental): Guam District Curriculum Guide, Howard County Common Core Math, learnzillion.com,Read Tennessee, 3rd Grade Math Flipbook

Reflection:

Content: Math	**Grade/Course**: 3rd Grade	**Timeline**: Week 7

Standard(s):
3.NF.3.a Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. a. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.
3.NF.3.b Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. b. Recognize and generate simple equivalent fractions, e.g., 1/2 = 2/4, 4/6 = 2/3). Explain why the fractions are equivalent, e.g., by using a visual fraction model.
3.NF.3.c Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram.
3.NF.3.d Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model

Lesson Overview:
These standards call for students to use visual fraction models (area models) and number lines to explore the idea of equivalent fractions and comparing fractions. It is important that they are able to justify their reasoning. Students should only explore equivalent fractions using models, rather than using algorithms or procedures.

Lesson Objective(s):
I Can Statements:
- I can use models to show and explain equivalent fractions.
- I can locate equivalent fractions on a number line.
- I can use models to show and explain whole numbers as fractions.
- I can locate whole numbers as fractions on a number line.
- I can use models to compare two fractions and record the comparison using <, >, or =.
- I can explain how the size of equal parts can be used to compare two fractions with the same numerator, and explain how the number of equal parts can be used to compare two fractions with the same denominator.

Vocabulary: compare, equivalent/equivalency, numerator, plot, whole, denominator, equal parts, equivalent, fraction, number line, numerator, visual fraction model, whole number, greater than (>), less than (<)

Focus Question(s):
What methods or strategies will I use to show the equivalency of a fraction?
When are two fractions equivalent?
What does it mean for two fractions to be equivalent and how would you show this to be true?

Description of Lesson (Including Instructional Strategies):
Anticipatory Set:
Draw a number line from 0 to 1. Put a tick mark in the middle of the number line and ask the students what fraction is shown by the tick mark. (Students should be able to say ½) Write ½ under the tick mark and ask the students to explain why it is ½.

Under your first number line, draw another number line exactly the same length. Put tick marks to divide the number line into fourths. Have the students identify the fractions shown. (¼ 2/4 ¾) "Looking at the two number lines, what two fractions are equivalent or the same amount? (1/2 and 2/4). How do you know?" (They are exactly the same length on the number lines.)

Set the objectives by reading your I Can Statements on equivalent fractions.

Instructional Strategies:

- **Step-by-Step Web Demo Lessons:** **https://learnzillion.com/lessons/1731-identify-equivalent-fractions-using-fraction-models**

- Web Resource on partitioning and understanding fractions: http://arb.nzcer.org.nz/supportmaterials/maths/concept_map_fractions.php#Partitioning

Have students explore several important ideas about these concepts in these standards:

- An important concept when comparing fractions is to look at the size of the parts and the number of the parts. For example, 1/8 is smaller than 1/2 because when 1 whole is cut into 8 pieces, the pieces are much smaller than when 1 whole is cut into 2 pieces.
- Students should recognize how to write whole numbers as fractions. The concept relates to fractions as division problems, where the fraction 3/1 is 3 wholes divided into one group. This standard is the building block for later work where students divide a set of objects into a specific number of groups. Students must understand the meaning of a/1. Example: If 6 brownies were shared between 2 people, how many brownies would each person get?
- Students can compare fractions with or without visual fraction models including number lines. Experiences should encourage students to reason about the size of pieces, the fact that 1/3 of a cake is larger than 1/4 of the same cake. Since the same cake (the whole) is split into equal pieces, thirds are larger than fourths.
- Students should also reason that comparisons are only valid if the wholes are identical. For example, 1/2 of a large pizza is a different amount than 1/2 of a small pizza. Students should be given opportunities to discuss and reason about which 1/2 is larger.
- Students build upon previous ideas from earlier grades to compare fractions with the same denominator. They see that for fractions that have the same denominator, the underlying unit fractions are the same size; wholes have been divided into the same number of equal parts. So the fraction with the larger numerator, or more of the same size parts, is the larger fraction. For example: 2/5 < 4/5 the fraction with the greater numerator is greater because it is made of more unit fractions. For another example, the segment on a number line from 0 to 3/4 is shorter than the segment from 0 to 5/4 because it measures 3 units of 1/4 as opposed to 5 units of 1/4, therefore 3/4 < 5/4.
- For unit fractions, students also see the one with the larger denominator is smaller, by reasoning, for example, that in order for more (identical) pieces to make the same whole, the pieces must be smaller. From this they reason that for fractions that have the same numerator, the fraction with the smaller denominator is greater. For example, 2/5 > 2/7, because 1/7 < 1/5, so 2 lengths of 1/7 is less than 2 lengths of 1/5. As with equivalence of fractions, it is important in comparing fractions to make sure that each fraction refers to the same whole.

Guided Practice: (on Personal White Boards)

- Have students model two fractions that are equivalent using number lines and shapes.

Formative Assessment:

- Worksheets for practice
- Daily Quick Checks

Independent Practice:

- Worksheets and Performance Tasks (Howard County Third Grade Math)

Closure:
Lesson Debrief:
Answer the focus questions at the end of the lesson.

Accommodations/Modifications:
Provide students with pairs of shapes or number lines that are already pre-partitioned. Provide concrete models or visuals.

Resources (Textbook and Supplemental): Guam District Curriculum Guide, Websites: Howard County Common Core Math, Read Tennessee Third Grade Math, www.learnzillion.com

Reflection:

Content: Math	**Grade/Course**: 3rd Grade	**Timeline**: Week 8

Standard(s):

3.NF.3.c Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram.

3.NF.3.d Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model

Lesson Overview:
These standards call for students to use visual fraction models (area models) and number lines to explore the idea of equivalent fractions and comparing fractions. It is important that they are able to justify their reasoning. Students should only explore equivalent fractions using models, rather than using algorithms or procedures.

Lesson Objective(s):
I Can Statements:

- I can use models to show and explain whole numbers as fractions.
- I can locate whole numbers as fractions on a number line.
- I can use models to compare two fractions and record the comparison using <, >, or =.
- I can explain how the size of equal parts can be used to compare two fractions with the same numerator, and explain how the number of equal parts can be used to compare two fractions with the same denominator.

Vocabulary: compare, equivalent/equivalency, numerator, plot, whole, denominator, equal parts, equivalent, fraction, number line, numerator, visual fraction model, whole number, greater than (>), less than (<)

Focus Question(s):
What methods or strategies will I use to show the equivalency of a fraction?
When are two fractions equivalent?
What does it mean for two fractions to be equivalent and how would you show this to be true?

Description of Lesson (Including Instructional Strategies):

Anticipatory Set:
(See link on Lesson Plans on *learnzillion* for excellent ideas for anticipatory set.)

Set the objectives by reading your I Can Statements on equivalent fractions.

Instructional Strategies:

- **Step-by-Step Online Demo Lessons and Lesson Plans:**

  https://hcpss.instructure.com/courses/97/pages/3-dot-nf-dot-3-learnzillion-resources

Have students explore several important ideas about these concepts in these standards:

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Guided Practice: (on Personal White Boards)

- Have students model two fractions that are equivalent using number lines and shapes.

Formative Assessment:

- Worksheets for practice
- Daily Quick Checks

Independent Practice:

- Worksheets and Performance Tasks (Howard County Third Grade Math)

Closure:
Lesson Debrief:
Answer the focus questions at the end of the lesson.

Accommodations/Modifications:
Provide students with pairs of shapes or number lines that are already pre-partitioned. Provide concrete models or visuals.

Resources (Textbook and Supplemental): Guam District Curriculum Guide, Websites: Howard County Common Core Math, Read Tennessee Third Grade Math, www.learnzillion.com

Reflection: